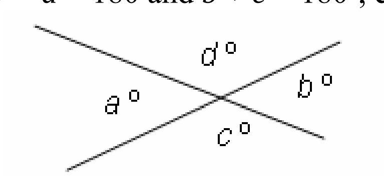
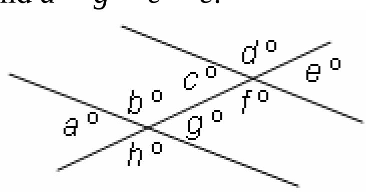
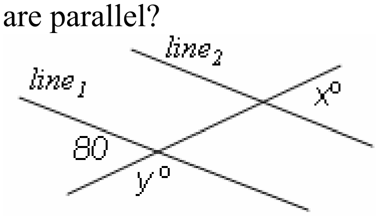
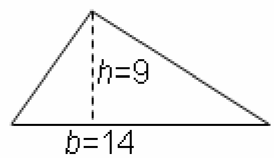
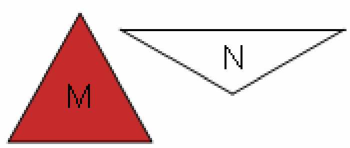
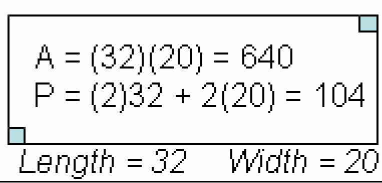
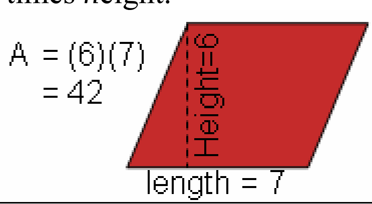
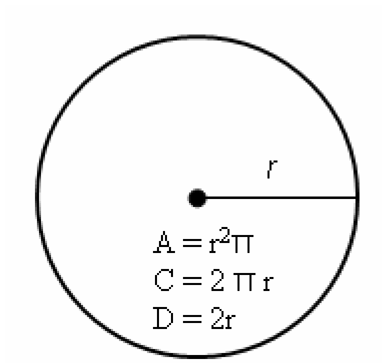
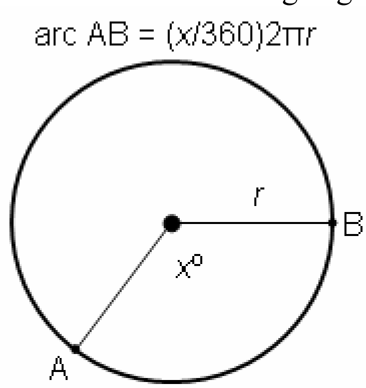
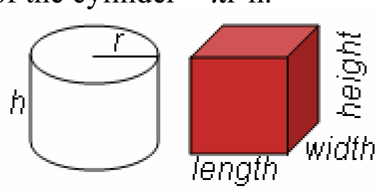
Basic Number Operations		
Concept	Explanation	Example Question / Use of Concept
Integers	Positive and Negative whole numbers, including zero. For example, -4, -3, -2, 1-1, 0, 1, 2, 3, 4, … are all integers.	**Q.** How many integers are greater than -5 and smaller than 3? **A.** 7. The integers are -4, -3, -2, 1, 0, 1, 2
Rational Numbers	Any number that can be expressed as a ratio. For example, 3.5, 22/6, or -.09.	**Q.** What is three-fourths times two-thirds? **A.** (3/4)(2/3) = (6/12) = (1/2) = 0.5.
Real Numbers	Integers, rational numbers, and those numbers that cannot be expressed as a fraction, such as π and √3.	**Q.** What is the square root of 16? **A.** 4, because 4^2=16
Order of Operations	In a complex expression, always perform the calculations inside the parentheses first, then exponents, then multiplication, then division, then addition, and finally subtraction.	**Q.** What is $((3^2-1) \times 2) + 4$ **A.** $((3^2-1) \times 2) + 4 = ((9-1) \times 2) + 4$ $((9-1) \times 2) + 4 = (8 \times 2) + 4$ $(8 \times 2) + 4 = 16 + 4$ 20
Arithmetic Sequence	In an arithmetic sequence, each term is equal to the previous term plus some constant *c*. For example, if x_i denotes the i^{th} item in the sequence, and x_1=4 and *c*=3, then x_2=7, x_3=10, and x_4=13.	**Q.** If in an arithmetic sequence the first number is 3, the constant factor is 4, then what is the 7^{th} number in the sequence? **A.** 21, because the sequence is 3, 6, 9, 12, 15, 18, and 21.
Geometric Sequence	In a geometric sequence, each term is equal to the previous term times some constant *m*. For example, if x_i denotes the i^{th} item in the sequence, and x_1=4 and *m*=2, then x_2=8, x_3=16, and x_4=32.	**Q.** What is the missing number in the geometric sequence 243, 81, 27, 9, ___? **A.** 3, because x_1=243, and x_2=81, so *m* = 1/3, and thus 9 times (1/3) = 3.
Factorization	The factors of a number are all of the integers that divide into that number with no remainder. For example, the factors of 27 are 1, 3, 9, and 27.	**Q.** What is the sum of all the factors of 12? **A.** 28, because the factors of 12 are 1, 2, 3, 4, 6 and 12, which sums to 28.
Multiples	The multiples of a number *x* are all of those numbers that can be divided by *x* without a remainder. For example, the multiples of 14 and 14, 28, 42, 56, 70, ...	**Q.** How many multiples of 23 are less than 100? **A.** Four, because the multiples of 23 are 23, 46, 69, 92, 115, 138, etc.
Odd, Even & Prime Numbers	Odd numbers are those which are not divisible by 2; Even numbers are multiples of 2, and prime numbers have only two factors, 1 and themselves. The first few odd numbers are 1, 3, 5, 7; the first few even numbers are 2, 4, 6, 8, 10; the first ten positive prime numbers are 1, 3, 5, 7, 11, 13, 17, 19, 23, and 29.	**Q.** How many numbers greater than 10 and less than 40 are odd and not prime? **A.** Six, because the odd numbers less than 40 and greater than 10 are {11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39}, and {15, 21, 25, 27, 33, 37} are not prime, because (3)(5)=15, (3)(7)=21, (5)(5)=25, (3)(9)=27, (3)(11)=33, etc.

Percentages, Averages, Means, Counting, and Basic Statistics & Probability		
Concept	Explanation	Example Question / Use of Concept
Percentages	"Percent" means "out of 100". For example, 25 percent means (25/100), or (1/4), so 25% of 12 apples is 3, because 12(1/4) = 3. Conversions using percentages can be done using the formula *part* = (*percent*/100) x *whole*	**Q1.** What is 25% of 300? **A1.** (25/100)300 = (1/4)300 = 300/4 = 75. **Q2.** 23 is what percent of 92? **A2.** 25%, because using the formula, 23 = (percent/100) x 92, we get percent = (23)(100)/92 = 25.
Average	Average = Sum of all the terms divided by the number of terms. So if the terms are 3, 4, and 8, the average is 13/3 = 5. Another name for "average" is "mean".	**Q.** If the average of the four terms 2, 3, 5 and *x* is 4, then what is *x*? **A.** x = 6. Using the formula, (10+*x*) /4=4, so 10 + *x* = 16, and *x* = 6.
Median & Mode	The median is the middle value of a list of numbers ordered by size. The mode is the number in a list that appears most often. For example, the median of {3,4,5,6,6,6} is 5, while the mode is 6.	**Q.** What is the median minus the mode of the numbers 7, 4, 7, 5, 7 and 6? **A.** -1. First, arrange the numbers in order, to get 4, 5, 6, 7, 7, 7. The median is 6, and the mode is 7, so 6-7=-1.
Counting & Probability	If an event *x* happens with probability *p*, and event *y* happens with a probability *q*, independent of event *x*, then the probability of events *x* and *y* happening is *p* times *q*. For example, if the probability of choosing a red apple out of bucket is (1/3), and the probability of choosing a rotten apple out of a bucket is (1/5), then the probability of selecting a red rotten apple is (1/3)(1/5) = 1/15.	**Q.** A deck has 20 cards. Each card is either red or blue, and each card has a triangle, a square, or a circle. If 5 of the cards are blue and 5 of the cards do not have a diamond, then what is the probability of selecting a card that is red and has a triangle or a square? **A.** 225/400, or 45/80.There must be 15 cards that are red, and 5 of the cards are either a square or a triangle. Thus, the chance of selecting a red card is 15/20, and the chance of selecting a card that is a square or a diamond is 15/20, and the chance of both happening is (15/20)(15/20) = 225/400.

Power, Exponents, Roots and Factoring		
Concept	Explanation	Example Question / Use of Concept
Bases, Exponents & Powers	The **base** of an expression is the number that is being manipulated, while the **exponent** is the value to which a number is being "raised" to. For example, in x^a, x is the base, while a is the exponent, and it means that x is multiplied a times, so 2^3 = (2)(2)(2) = 8. A "power" of a base is the exponent of that base.	
Zero power and negative exponents	Any number raised to the zero power is 1. A number with a negative exponent is equivalent to 1 divided by that number raised to the positive exponent: $x^0 = 1$ $x^{-a} = 1/(x^a)$	**Q.** What is 3.1432234 raised to zero-th power? **A.** 1, by definition. **Q.** What is $2^{(-2)}$? **A.** 0.25, because $2^{(-2)} = 1/(2^2) = ¼ = 0.25$.
Exponents with Addition and Subtraction	Numbers that have the same base and exponent can be summed together. For example, $x^3 + 2x^3 – 7x^3 = -4x^3$.	**Q.** Simplify $x^3 + 2y^7 – 7x^7 + x^3 + 3y^7 – 4x^3$ **A.** $-2x^3 + 5y^7 – 7x^7$.
Exponents and Multiplication	If two numbers of the same base but with different exponents are multiplied, then the exponents can be added: $n^a \times n^b = n^{(a+b)}$	**Q.** What is $(2^1)(2^2)(2^3)$? **A.** 64, because $(2^1)(2^2)(2^3) = (2^{1+2+3}) = 2^6$.
Exponents and Division	If dividing two numbers of the same base, subtract the exponent of the denominator from the exponent of the numerator: $n^a / n^b = n^{(a-b)}$	**Q.** What is three to the four-thirds divided by three to the one-third? **A.** 3, because $3^{(4/3)}/3^{(1/3)} = 3^{(4/3 – 1/3)} = 3^1$ **Q.** What is $4^{24}/4^{22}$? **A.** 16, because $4^{24}/4^{22} = 4^{(24-22)} = 4^2 = 16$.
Exponents and Powers	An exponent raised to some power is the same as the product of the power and the raise: $(n^a)^b = n^{(ab)}$	**Q.** What is $(3^4)^{(1/2)}$? **A.** 9, because $(3^4)^{(1/2)} = 3^{(4(1/2))} = 3^2 = 9$.
Exponents and Products	The product of two numbers raised to some exponent can be rewritten as a product of the individual numbers: $(nm)^a = n^a \times m^a$	**Q.** What is $((3)(4))^2$? **A.** 144, because $((3)(4))^2 = (3^2)(4^2)) =$ (9)(16).

Variables, Equations, Substitution, Factoring, Functions and Lines.		
Concept	**Explanation**	**Example Question / Use of Concept**
Variables & Equations	A variable is a symbolic representation used to denote a quantity or expression. An equation is an expression that utilizes variable. For example, in the equation $4x = 12$, the variable is x, and in this particular case, when $x = 3$, then the equation is satisfied.	**Q1.** If x = 3 and $y = 2$, then what is the value of $3x^2 – 2y$? **A1.** 23, because the equation then is $3(3)^2 – 2(2) = 3(9) – 4 = 27 – 4 = 23$. **Q2.** What value of x satisfies $3x^2 - 6x = 0$? **A2.** 2, because $3(2)^2-6(2) = 3(4)-12 = 0$.
Substitution	Substitution is used to solve equations. If there are two equations with two variables, solve for one of the variables, and then use the result to solve for the second variables.	**Q.** If $3x + y = 9$, and $2x – 2y = -2$, then what is the value of $x + y$? **A.** Using the first equation, $y = 9 - 3x$, and substituting that into the second equation, we have $2x – 2(9-3x) = -2$, so $x = 2$.
Factoring Equations	Factoring an equation is a rewriting of the equation, with several "common" rules: $a^2 + 2ab + b^2 = (a + b)(a + b)$ $a^2 - 2ab + b^2 = (a - b)(a - b)$ $a^2 - b^2 = (a + b)(a - b)$	
Functions	A function is a rule that transforms one number x into another number y, and is written as $y = f(x)$, where f is the function.	**Q.** If $x@y = x+y$, then what is 4@16? **A.** 20. You are told that @ is a function that, given two variables, adds them, so 4@16 = 4+16 = 20.
Intersecting Lines	Opposite angles of intersecting lines are equal; the angles along the same line add to 180 degrees. Here, a = b, d = c, and $b + d = 180$ and $b + c = 180°$, etc. 	**Q.** Two lines intersect to form 4 angles. If one of the angles formed is 34°, then what is the sum of the other 3 angles? **A.** 326. When two lines intersect, two pairs of equal angles are formed. If one of the angles is 32°, the other angle which forms the straight line is 180-32=148°. Thus, the other three angles formed are 146, 146, and 34, so 146+146+34 = 326.
Parallel Lines	When a line crosses two parallel lines, eight angles are formed, with two sets of four angles being equal. Here, $d = f = b = h$, and $a = g = c = e$. 	**Q.** What is the value of $x + y$ if lines 1 and 2 are parallel?  **A.** 180, because $y = 100$, and $x = 80$, so $x + y = 180$.

Triangles		
Concept	**Explanation**	**Example Question / Use of Concept**
Area and interior angles of a Triangle	The area of a triangle is one-half base times the height: $A = ½\, b\, h$, and the three interior angles of a triangle add up to 180 degrees. A = ½ (14)(9) = 63.  	**Q.** If the area of a triangle *M* is 32 and the height is 32, then what is the base of *M*? **A.** 2, because using the, we have *b* = 32(*h*)(2/32) = 2. **Q.** If triangle *N* has angles 33 and 74, then what is the third interior angle? **A.** 73, because the interior angles of a triangle add to 180 degrees.
Equilateral and Isosceles Angles	An equilateral triangle has 3 equal sides and angles, while an isosceles triangle has 2 equal sides and angles. Triangle M is equilateral and so each angle is 60º and the sides are equal, while isosceles triangle N has two equal angles.  	**Q.** If triangle *P* is isosceles, and one of the angles is 78 degrees, then what is the smallest possible value of any of the three angles? **A.** 14º. Either 78 is one of the two equal angles, or it is not. If 78 is one of the two equal angles, then the second angle is 78, and so the third angle is 180-78-78=14. If 78 is not one of the equal angles, then the other angles are each 180-78=102/2=51º, and so 14 is smaller than 51.
Rectangles	The area (A) of a rectangle is *l*ength times *w*idth. The perimeter (P) of a rectangle is 2*l* + 2*w*. Opposite sides of rectangles are equal in length. A square is a rectangle where all of the sides are of equal length.  	**Q.** If a rectangle has perimeter 22, and one of the sides has length 7, then what is the area of the rectangle? **A. R**ectangles have equal opposite sides, so two of the sides are 7, and the remaining two sides sum to 22 – 2(7) = 8, and so each side is 8/2 = 4. The area is (7)(4) = 28.
Parallelograms	The area (A) of a parallelogram is *l*ength times *h*eight.  	**Q.** If the length of parallelogram *N* is 32, and the height is one fourth of the length, then what one-half the area of *N*? **A.** 128. The height is one-fourth of 32, or 8, so the area is (8)(32) = 256. One-half of 256 = 256/2 = 128.

Circles and Solid Geometry		
Concept	**Explanation**	**Example Question / Use of Concept**
Area, Radius, Diameters and Perimeter and circumference of Circles	For a circle, the radius (*r*) is the distance from the center to the edge; the area (*A*) is π times the radius squared; the diameter (*D*) is 2*r*; the circumference (*C*) of a circle = 2 π times *r*.  	**Q1.** If the area of a circle is 36π, then what is the diameter of the circle? **A1.** 12. Using the formula for the area of a circle, the radius of the circle must be the square root of 36, or 6, and the diameter is twice the radius, so the diameter is 12. **Q2.** The diameter of a circle is 128. What is the circumference of the circle? **A2.** Using the formula for the diameter of the circle, the radius is 64. Thus, the circumference is $2\pi r$, or $2\pi(64) = 128\pi$.
Arcs of a Circle	An arc of a circle is a segment of the perimeter; the length of the arc is 2π times *r* times the intersecting angle (*x*).  	**Q.** A circle has radius 2. If the intersecting angle of an arc of the circle is 72 degrees, then what is the length of the arc? **A.** $4\pi/5$. Using the formula, the length of the arc is $(72/360)(2)(\pi)(2) = 4\pi/5$. **Q.** The length of an arc of a circle is 16π and the intersecting angle is 180 degrees, then what is the diameter *D* of the circle? **A.** 16. Using the formula, arc = $16\pi = (180/360)(\pi)(r)$, and so $r = (16\pi)(360)/(\pi)(180) = 16/2 = 8$, and the diameter is $2r$, so $D = (8)(2) = 16$.
Solid Geometry	The volume of a rectangular box is *l*ength times *w*idth time *h*eight. The volume of a cylinder is base times *h*eight, where the base is a circle, so the area of the cylinder = $\pi r^2 h$.  	**Q.** If a cylinder has volume $27\pi cm^3$, and the height is equal to the radius, then what is the radius of the cylinder? **A.** 3. Using the formula, $27\pi = \pi r^2 h$, and because $r=h$, then $27\pi = \pi r^3$, and so $27 = r^3$, so $r = 3$. **Q.** What is the volume of a cube where the *l*ength, *w*idth, *h*eight are each 4cm? **A.** Volume of box = $lwh = 4^3 = 64cm^3$.